Federal University of Santa Catarina
Graduate Program in Engineering and Mechanical Sciences

Plasmas and electrical discharges in gases

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SUMMARY
Plasmas and electrical discharge in gases

- Kinetic theory of gases
- Atomic structure
- Ionization
- Deionization
- Electron emission
- Behavior of charged particles in a gas in electric fields of low E/p
- Behavior of charged particles in a gas in electric fields of high E/p
- Glow discharges
- Plasmas

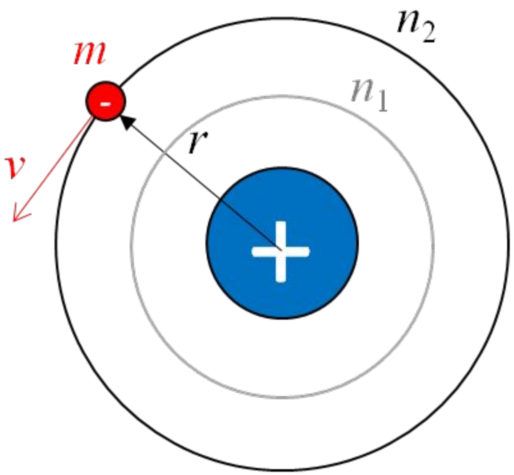
ATOMIC STRUCTURE
Bohr model - postulates

- One of the most simple models for the atomic structure is based on the following two postulates:

https://pt.wikipedia.org/wiki/Niels_Bohr

First postulate:

"The electron can exist only in certain stable orbits":



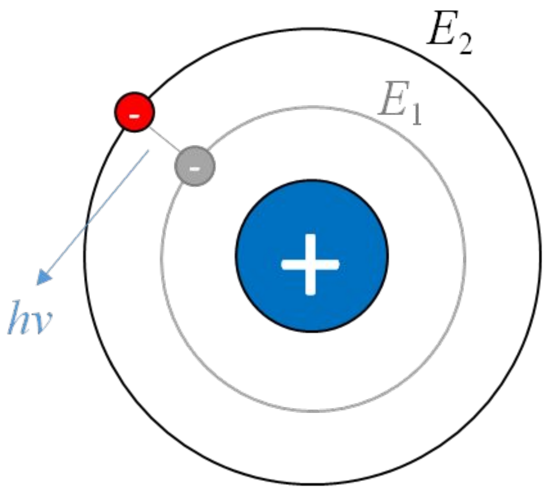
$$mvr = \frac{nh}{2\pi} \quad (1)$$

where m is the electron mass, v the orbital speed, r the orbital radius, n the primary quantum number (= 1, 2, 3, ...) and h the Planck constant (6.63×10^{-34} J/s).

ATOMIC STRUCTURE
Bohr model - postulates

Second postulate:

"When the energy of an atom changes from a value E_2 *to a lower value* E_1 *the difference in energy is emitted as a quantum of radiation whose frequency is given by the relation:*



$$h\nu = E_1 - E_2 \quad (2)$$

Similarly, the electron energy increases from E_1 to E_2 after absorption of external radiation. This process describes a particle excitation or ionization.

ATOMIC STRUCTURE
Bohr model - orbital radius

- The orbital radius is given by:

$$F = \frac{e(eZ)}{4\pi\epsilon_0 r^2} = \frac{mv^2}{r} \therefore \boxed{r = \frac{e^2 Z}{4\pi\epsilon_0 m v^2}}$$

where e is the fundamental charge and Z the atomic number (number of protons). Replacing the first postulate in equation 3 we obtain the orbital radius of the n-th quantum shell:

$$r = \frac{e^2 Z}{4\pi\epsilon_0 m v^2} = \frac{e^2 Z}{4\pi\epsilon_0 m\left(\frac{nh}{2\pi m r}\right)^2} \therefore \boxed{r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2 Z}} \tag{3}$$

ATOMIC STRUCTURE
Bohr model - orbital radius

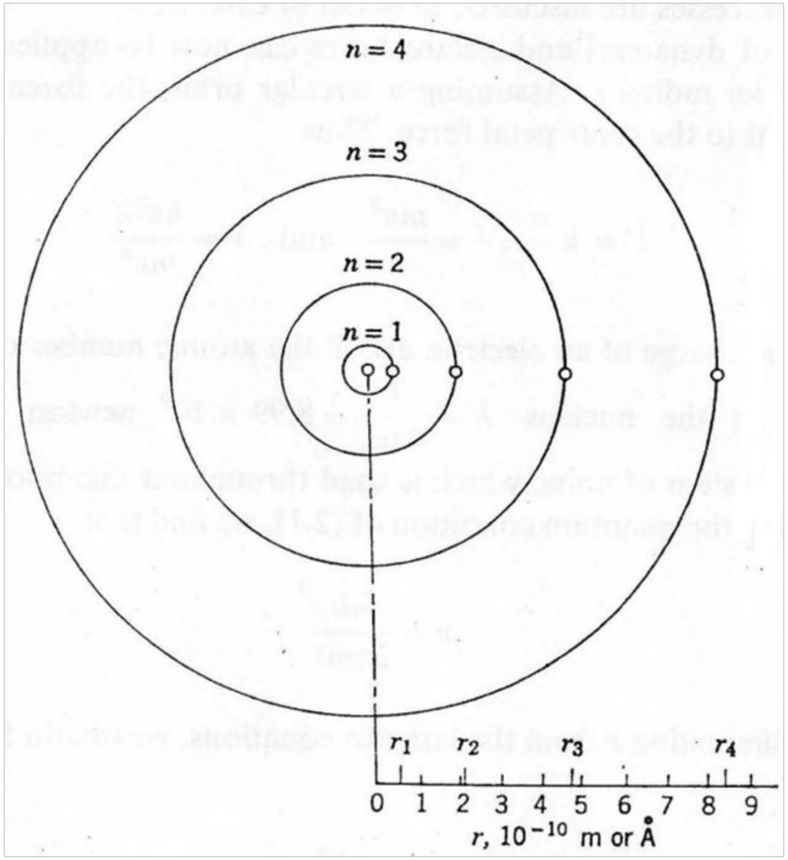
- Replacing the constants:
  - $\varepsilon_0 = 8.85\times10^{-12}\ \mathrm{C^2/N{\cdot}m^2}$
  - $m = 9.11\times10^{-31}$ kg
  - $e = 1.6\times10^{-19}$ C

we get

$$\boxed{r_n = \frac{\epsilon_0 n^2 h^2}{\pi m e^2 Z} = 5.31 \times 10^{-11}\frac{n^2}{Z}}$$

- The orbital radius of an electron at the fundamental state is $r = 0.531\times10^{-10}$ m. This result shows that the diameter of the H atom is around 1×10^{-10} m (1 Å).

ATOMIC STRUCTURE
Bohr model - orbital radius



First four orbits of the H atom.

E. Nasser, Fundamentals of Gaseous Ionization and Plasma Electronics, New York: Wiley, 1971

ATOMIC STRUCTURE
Bohr model - kinetic energy

- The kinetic energy of an electron in orbital motion at the n-th quantum shell is given by:

$$K_n = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\frac{nh}{2\pi mr}\right)^2 = \frac{n^2h^2}{8\pi^2mr^2} \tag{4}$$

- Replacing equation 3 in equation 4:

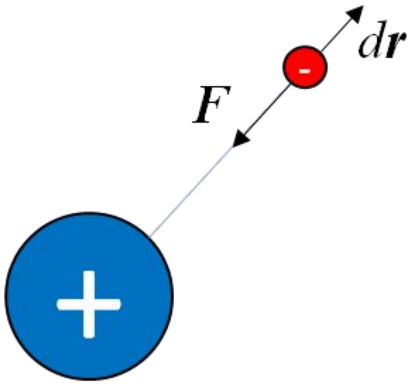
$$K_n = \frac{n^2h^2}{8\pi^2m\left(\frac{\epsilon_0^2n^4h^4}{\pi^2m^2e^4Z^2}\right)} \therefore \boxed{K_n = \frac{me^4Z^2}{8\epsilon_0^2n^2h^2}} \tag{5}$$

- K_n increases as the primary quantum number decreases. If the atom is at rest, K_n is also its total kinetic energy.

ATOMIC STRUCTURE
Bohr model - potential energy

- The stored potential energy is given from the work done on the electron by the electrical force produced by the atom nucleus:

$$U = -\int_{\infty}^{r} \vec{F} \cdot d\vec{r} = \int_{\infty}^{r} \left[\frac{e(eZ)}{4\pi\epsilon_0 r^2} \right] dr$$



$$U = \frac{e^2 Z}{4\pi\epsilon_0} \int_{\infty}^{r} \frac{dr}{r^2} \therefore \boxed{U = -\frac{e^2 Z}{4\pi\epsilon_0 r}} \qquad (6)$$

- Replacing equation 3 in equation 6 we get the potential energy stored by the electron at the n-th quantum shell:

$$U_n = -\frac{e^2 Z}{4\pi\epsilon_0 \left(\frac{\epsilon_0 n^2 h^2}{\pi m e^2 Z} \right)} \therefore \boxed{U_n = -\frac{m e^4 Z^2}{4\epsilon_0^2 n^2 h^2}} \qquad (7)$$

ATOMIC STRUCTURE
Bohr model - total energy

- The negative sign assigned to the potential energy means that the electron is in a "bound state". The total electron energy at the n-th quantum shell is given from equations 5 and 7:

$$E_n = K_n + U_n = \frac{me^4Z^2}{8\epsilon_0^2n^2h^2} - \frac{me^4Z^2}{4\epsilon_0^2n^2h^2} = -\frac{me^4Z^2}{8\epsilon_0^2n^2h^2} = -\frac{E_i}{n^2}$$

where $E_i = me^4Z^2/8\varepsilon_0{}^2h^2$ is the ionization atom energy. For an H atom, the ionization energy, *i.e.*, the energy required to unbound the electron from the nucleus electric field is given by:

$$E_i = -\frac{me^4Z^2}{8\epsilon_0^2h^2} = -2.16 \times 10^{-18} J = -13.6 \text{ eV}$$

ATOMIC STRUCTURE
Bohr model - photon emission

- Consider a transition between quantum states, from n_1 to n_2. When the electron come back to the fundamental state, the energy absorbed in the excitation process is released as radiation:

$$h\nu = E_2 - E_1 = -\frac{me^4Z^2}{8\epsilon_0^2 n_2^2 h^2} + \frac{me^4Z^2}{8\epsilon_0^2 n_1^2 h^2} = \frac{me^4Z^2}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

- The radiation is an electromagnetic wave travelling at the speed of light c with wavelength λ:

$$\frac{hc}{\lambda} = \frac{me^4Z^2}{8\epsilon_0^2 h^2}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \therefore \boxed{\frac{1}{\lambda} = \frac{me^4Z^2}{8\epsilon^2 ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)} \qquad (8)$$

Bohr model - photon emission

- Equation 8 can be rewritten as:

$$\frac{1}{\lambda} = \frac{me^4Z^2}{8\epsilon^2ch^3}\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) = RZ^2\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \qquad (9)$$

where $R = 1.09\times10^7$ m^{-1} is the Rydberg constant. Equation 9 presents a direct relation between the radiation wavelength emitted by an atom with the distance between the transition shells.

- For a transition from $n_2 = 6$ to $n_1 = 2$, equation 9 gets:

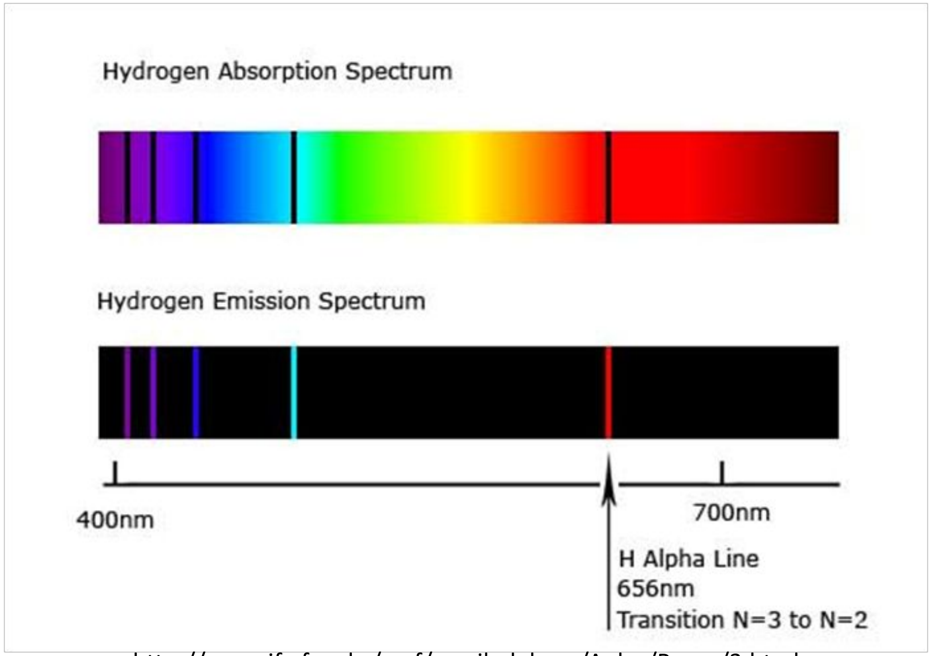
$$\frac{1}{\lambda} = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(6)^2}\right] = \frac{2}{9}R$$

ATOMIC STRUCTURE
Bohr model - photon emission

that means $\lambda \sim 410$ nm (visible – violet). For a transition from $n_2 = 3$ to $n_1 = 2$:

$$\frac{1}{\lambda} = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = \frac{5}{36}R$$

we get $\lambda \sim 660$ nm (visible – red).



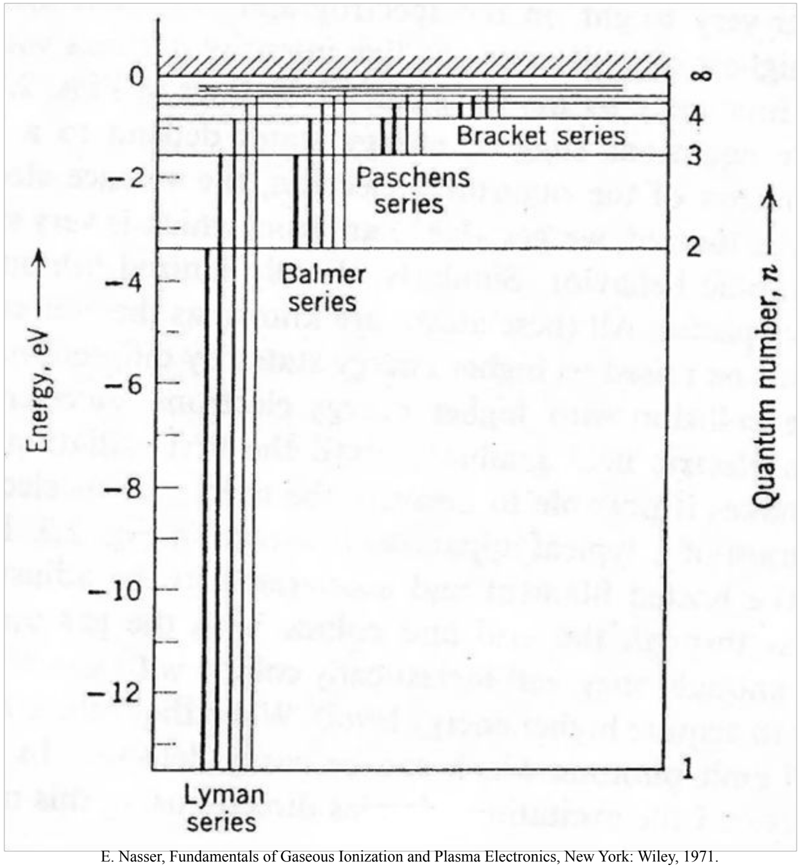
http://www.if.ufrgs.br/cref/camiladebom/Aulas/Pages/3.html

$\lambda \sim 660$ nm (H_α)

$\lambda \sim 410$ nm (H_δ)

ATOMIC STRUCTURE
Bohr model - photon emission

- The H spectral lines are described by Lyman, Balmer, Paschen e Bracket series:



E. Nasser, Fundamentals of Gaseous Ionization and Plasma Electronics, New York: Wiley, 1971.

<u>Lyman</u>: transitions to $n = 1$

<u>Balmer</u>: transitions to $n = 2$

<u>Paschen</u>: transitions to $n = 3$

<u>Bracket</u>: transitions to $n = 4$

ATOMIC STRUCTURE
Reading

- Chapter 2 - E. Nasser, Fundamentals of Gaseous Ionization and Plasma Electronics (pages 24-32).

See you next topic!

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